UNIFYING STUDY OF BASIC PROBLEMS
IN ENGINEERING AND PHYSICAL SCIENCES
BY MEANS OF GEOMETRY

RAAG
Research Notes

Third Series No. 7/5

On the Continuous Communication Networks
including Feedback Branches

by

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December 1963

RESEARCH ASSOCIATION OF APPLIED GEOMETRY

應用幾何學研究協會
フィードバック枝のある通信回路網について

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＜概要＞

フィードバック枝を含んだ離散的な通信系については、
早くから Shannon [1]によって考察されており、フィードバッ
ク枝の存在は系の通信容量をいささかも増加させないこ
とが解っている。従って離散的通信回路網の通信容量につ
いては、フィードバック枝を無視して単なる最大流量問題
として扱えるので、すでに多くの研究がなされている [2],
[3] 。一方連続通話路にフィードバック枝の加わった場合の
研究は未だほとんどされておらず、最近では離散的な場
合の Shannon の説を無制限に拡張して、やはり前方方向の
容量は変化せず、ただ符号化が簡単になるだけであるとい
う誤解すら生じているようである [4]。

ここではフィードバックを含んだ一般の連続通信回路網
について、回路の各点でのその点への入力信号の線型結合
をとるような方式を採用する際の回路網の構造を調べる。
これによってフィードバック枝の与える影響のみならず、
回路中の一つの枝の向きを逆転した時の影響がわかり、こ
の種の回路の構造が明らかになる。又各端子での最適信号
決定方式も決定できる。

この研究は回路網理論に一つの新しいモデルを加えるもの
であり、又信号空間の刻接 [6] の一つの応用ともみることができ
る。
On the Continuous Communication Networks including Feedback Branches

by

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Summary. A network consisting of continuous communication channels is treated under the following condition. At each node of the network, the input signals to the node are linearly combined and then it is transmitted in the branches directed away from the node. The noises of the branches are added to the signals flowing in them.

The structure of the network of this kind is clarified and the combining ratios of the signals at each node, by which the output signal-to-noise (SN) ratio is maximized, are obtained for given SN ratios of the branches. Moreover, it is proved that the optimal SN ratio does not change when a branch is replaced by a branch having the same characteristic but different orientation. By this theorem, a feedback branch is clarified to play a similar role as it is connected in the forward direction, in a remarkable distinction from the discrete case where a feedback branch does not increase the communication capacity of the forward direction.
1. Introduction. In many real communication systems, a number of communication channels work together in a network system including feedback channels. In the case of a discrete communication system, it has been proved that the existence of feedback branches never increases the total communication capacity of the system, but only simplifies the coding scheme [1]. Therefore, the capacity of a discrete communication network can be obtained as the maximum amount of flow through the network, the capacities of whose branches are given, ignoring the feedback branches. Such problems have been fully investigated, e.g., in [2], [3] etc.

However, the analysis of communication networks has hitherto been limited to discrete ones), and continuous communication networks have scarcely been investigated. Even the influence of a feedback channel added to the forward one cannot yet be clarified. Moreover, there seems to be some misunderstanding that a feedback channel does not increase the channel capacity of forward direction as well as in the case of discrete channels.

Here we shall investigate continuous communication networks including feedback branches under the following signal connexion at each node. We assume that the signals flowing into a node are linearly combined to yield the node signal which in turn flows into the branches directed away from the node. This condition is adopted not only because it is frequently used in practice and simplifies the problem, but also because it seems that this is the only way to increase the forward capacity using a feedback channel.

Complicated coding accompanies of necessity time-lag of signals. Hence, even if we obtain some information concerning noises added in the forward channels from the feedback channels, it requires an additional channel to correct or reduce the errors by the noises, if the feedback information is time-lagged as is the same in the case of discrete systems. Therefore, it seems that only the signal transmission with direct (no time-lag) combination at each node can reduce the noises instantly through the feedback branches.

Under the above conditions, we can clarify the structure of continuous communication networks. We can prove that the maximum attainable output SN ratio through the network does not change even if one of the branch is inverted2). Therefore,

1) See also [4].
2) Except the case when the derived network has the redundant nodes from which any branches do not issue or in which any branches do not enter.
we see that a feedback branch can perform the same role as the forward channel, if the combining ratio of signals is appropriately chosen. Moreover, we can solve the problem of finding the combining ratios of the nodes by which the output SN ratio is maximized.

It is also noted that the problem can be regarded as an application of the theory of Diakoptics of Information Spaces [5], [6]. In this case, the network can be regarded as a signal space having the dimensions of the input signal space multiplied by the number of the branches. The output signal is derived by the reduction of this space. To specify the combining ratios is to determine the direction of signal reduction in the space. The possible directions of signal reduction is limited by the topological structures of the underlying network.

From another point of view, this problem can be considered to give a new model to the topological network theory. The model will be located between the signal flow graph and the network of Linear Programming with Weak Graphical Representation given in [7]. Our problem of finding the optimal combining ratios can be regarded as the synthesis of the signal flow graph.

2. Problem. Let us consider a network consisting of oriented or directed branches and nodes incident to them. The branches represent communication channels. A branch directed away from a node is called an output branch of the node, and a branch directed toward a node is called an input branch of the node. There is an input node in the network. The original signal flows in the network through an input branch of the input node. There is also an output node. The final output signal is obtained from one output branch of the output node (see Fig. 1).

If a network has a node to which no input branches belong or no output branches belong, the node together with all the branches incident to it does not play any roles in the communication network. Hence, we call the network meaningless and pay no attention to such networks.

The characteristics of the nodes and branches of the network are stated as follows.
Branches. The branches represent a continuous communication channels. Each communication channel is disturbed by an additive Gaussian noise. When a signal (the original signal plus the noise added hitherto) \( x_k \) is flowing in branch \( \kappa \) (\( \kappa = 1, 2, \ldots, n \); \( n \) is the number of the branches), a noise \( n_k \) is newly added to it in this branch. The signal-to-noise ratio \( \rho_k \) of the branch is

\[
\rho_k = \frac{(x_k)^2}{(n_k)^2} ,
\]

where the bar means the averaged, and so \( (x_k)^2 \) and \( (n_k)^2 \) are the signal and noise powers, respectively. The noises appearing in the branches are assumed to be independent or to have no mutual correlations.

Nodes. At each node, the signals flowing in the input branches of a node are linearly combined, and then the combined signal flows into every output branch of the node. Let the combining ratio of the signals of the input branches of node \( a \) (\( a = 1, 2, \ldots, m \); \( m \) is the number of the nodes) be denoted by \( \beta_k^a \). Then the signal \( x^a \) which in turn will flow in the output branches of the node \( a \) can be written as

\[
x^a = \sum_k \beta_k^a x_k ,
\]

where \( x_k \) are the signals flowing in the branches \( \kappa \), and

\[
\beta_k^a = 0
\]

if branch \( \kappa \) is not an input branch of node \( a \).

Here we normalize the signals such that all the signals flowing in the branches have unit power. This can be done without loss of generality, since SN ratios \( \rho_k \) only are given as the characteristics of the branches. Then the sum of the combining ratios at a node is normalized to 1, i.e.,

\[
\sum_k \beta_k^a \rho_k = 1 ,
\]

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where $\beta^a_\kappa$ is equal to 1 if branch $\kappa$ is an input branch of node $a$, and equal to 0, otherwise.

The problem of our continuous communication network is stated as follows.

**Problem.** Given the branch characteristics (SN ratios of the branches), find the optimal combining ratio $\beta^a_\kappa$ of every node for which the final output SN ratio is maximized.

It should be noted that the SN ratios of all the nodes but of the output node cannot necessarily be maximized by the above optimal combining ratios $\beta^a_\kappa$. Hence, we cannot solve the problem in such a simple way of determining the combining ratio of every node independently.

3. Theorems representing the Fundamental Structures of the Network and Solution of the Problem. Let $n^\kappa$ be the noise added in branch $\kappa$. Then the noise $n$ appearing in the output signal must be expressed by their linear combination. Denoting the coefficients of the combination by $\alpha_\kappa$, we obtain

$$n = \sum_\kappa \alpha_\kappa n^\kappa. \tag{4}$$

We call the $\alpha_\kappa$ the attenuation coefficient concerning branch $\kappa$. The signal $x$ flowing in branch $\kappa$, which will arrive at the output node through various routes including feedback loops, will be attenuated to $\alpha_\kappa x$ until it arrives at.

We can easily calculate $\beta^a_\kappa$'s from $\alpha_\kappa$'s by the following lemmas. On the contrary, when $\beta^a_\kappa$'s are given, $\alpha_\kappa$'s will be calculated by the signal-flow-graph method.

**Lemma 1.** When branch $\kappa$ is an input branch of node $a,$

$$\beta^a_\kappa = \frac{\alpha_\kappa}{\sum_\kappa \beta^b_\kappa \alpha_\kappa}. \tag{5}$$

holds.

**Proof.** Once the signals flowing in the input branches of a node $a$ reach the node $a$, they are combined and go together.
thereafter. Hence, the differences between $\alpha_\kappa$ of the branches are due only to the differences between the combining factors $\beta_\kappa$ of the node $a$. Therefore, the $\alpha_\kappa$ are proportional to the $\beta_\kappa$, and we have

$$\beta_\kappa^a = \frac{\alpha_\kappa}{\sum_\lambda \beta_\lambda^a \alpha_\lambda},$$

because of

$$\sum_\lambda \beta_\lambda^a \alpha_\lambda = 1.$$

**Lemma 2.** When branch $\kappa$ is an output branch of node $a$,

$$\beta_\kappa^a = \frac{\alpha_\kappa}{\sum_\lambda \beta_\lambda^a \alpha_\lambda}$$

holds, where $\beta_\lambda^a$ is equal to 1 when the branch $\lambda$ is an output branch of the node $a$, and equal to 0 otherwise.

**Proof.** Let us consider the noises $n_\kappa$ produced in the input branches of node $a$. They reach the output node at last multiplied by the attenuation coefficients $\alpha_\kappa$ as

$$\sum_\kappa \beta_\kappa^a \alpha_\kappa n_\kappa.$$

From another viewpoint, the $n_\kappa$ are multiplied by $\beta_\kappa^a$ before they flow into the output branches of the node $a$. The noise $\sum_\kappa \beta_\kappa^a n_\kappa$ at the node $a$ enters into each of the output branches $\lambda$ of the node, and is multiplied by $\sum_\lambda \beta_\lambda^a \alpha_\lambda$ until it reaches the output, since the noise flowing in $a$ branch $\lambda$ is multiplied by $\alpha_\lambda$ until it reaches the output. Therefore, it must holds that

$$\sum_\kappa \beta_\kappa^a \alpha_\kappa n_\kappa = \left(\sum_\kappa \beta_\kappa^a n_\kappa\right) \left(\sum_\lambda \beta_\lambda^a \alpha_\lambda\right).$$
Comparing the coefficients of $a$ in the both sides of the equation, it holds that

$$\alpha_a^\kappa = \beta_a^\kappa \left( \sum_{\lambda} \overline{D}_{\lambda}^{a} \alpha_{\lambda}^{\kappa} \right).$$

and then

$$\beta_a^\kappa = \frac{\alpha_a^\kappa}{\sum_{\lambda} \overline{D}_{\lambda}^{a} \alpha_{\lambda}^\kappa}.$$

Since $\alpha_a^\kappa$'s are related to $\beta_a^\kappa$'s, they cannot take an arbitrary set of values but are restricted by the following lemma.

Lemma 3. The attenuation coefficients must satisfy the continuity condition of the Kirchhoff type,

$$\sum_{\kappa} \overline{D}_{\kappa}^{a} \alpha_{\kappa}^a = 0,$$

(7)

where $D_{\kappa}^{a}$ is the incidence matrix of the network, i.e.,

$$D_{\kappa}^{a} = \begin{cases} +1, & \text{if branch } \kappa \text{ is an input branch of node } a, \\ -1, & \text{if branch } \kappa \text{ is an output branch of node } a, \\ 0, & \text{otherwise.} \end{cases}$$

Proof. From the definitions of $\overline{D}_{\kappa}^{a}$, $D_{\kappa}^{a}$, and $D_{\kappa}^{a}$, it is easy to see

$$D_{\kappa}^{a} = \overline{D}_{\kappa}^{a} - D_{\kappa}^{a}$$

(8)

hold. From (5) and (6), we have

$$\sum_{\kappa} \overline{D}_{\kappa}^{a} \alpha_{\kappa}^a = \alpha_a^\kappa / \beta_a^\kappa,$$

-7-
\[
\sum_k \vec{D}_k \alpha_k = \alpha_k / \beta_k^a.
\]
Subtracting the latter from the former, we have

\[
\sum_k D^a_k \alpha_k = 0.
\]

The output noise can be written as

\[
n = \sum_k \alpha_k n^k,
\]
and the output noise power \( N \) is

\[
N = \sum_{k, \lambda} \alpha_k \alpha_{\lambda} \frac{n^k n^\lambda}{n^k n^\lambda} = \sum_k n^k \alpha_k^2,
\]
where \( N \) is the power of the noise \( n^k \), and the independence of \( n^k \) 's are taken into account. Since the signal flowing in the branches as well as at the output, are normalized to 1, the signal-to-noise ratios are respectively

\[
\rho_k = \frac{1}{N^k}, \quad \rho = \frac{1}{N},
\]
and it follows that

\[
\rho = \frac{1}{\sum_k \alpha_k^2 / \beta_k^2}
\]
which is the object to be maximized. Hence the problem of the continuous communication network can be stated in the following manner.

**Problem.** Given the SN ratios \( \rho_k \) of the branches, find the set of

\[
\alpha_k \text{'s satisfying } \sum_k D^a_k \alpha_k = 0, \text{ which minimize}
\]
\[
\frac{1}{\rho} = \sum_k \frac{\alpha_k^2}{\rho_k}
\]

Reformulating the problem as above, we can at once prove the following theorem which shows the fundamental structure of the network.

**Theorem 1.** Let \( X' \) be the network obtained from a network \( X \) by inverting the orientation of a branch \( \kappa \). If both networks are not meaningless, i.e. have no redundant nodes, the maximal output SNR ratios of the both networks are the same, i.e., they have the same ability of signal transmission.

**Proof.** Let us denote the quantities of \( X' \) by attaching the primes. The problem of \( X \) is to maximize

\[
\rho = \frac{1}{\sum_k \frac{\alpha_k^2}{\rho_k}}
\]

under the condition

\[
\sum_k D^a_k \alpha_k = 0,
\]

and that of \( X' \) is to maximize

\[
\rho' = \frac{1}{\sum_k \frac{\alpha'_k \rho'_k}{\rho_k}}
\]

under the condition

\[
\sum_k D'_k \alpha'_k = 0.
\]

Let us define \( \xi_k \) by

---

---
\[ \varepsilon_{\kappa} = \begin{cases} +1, & \text{if } \kappa \text{ is not the inverted branch,} \\
-1, & \text{if } \kappa \text{ is the inverted branch.} \end{cases} \]

Then, obviously
\[ D^a_K = \varepsilon_{\kappa} D^a_K. \]

Since \( \varepsilon_{\kappa}^2 = 1 \) always holds, the problem of \( X' \) can be reformulated as "Maximize
\[ \rho' = \frac{1}{\sum_{\kappa} (\varepsilon_{\kappa} \alpha'_{\kappa})^2} \]
under the condition
\[ \sum_{\kappa} D^a_K (\varepsilon_{\kappa} \alpha'_{\kappa}) = 0. \]

This problem is the same as that of \( X \), if we put \( \alpha'_{\kappa} = \varepsilon_{\kappa} \alpha_{\kappa}. \)
Hence we have, for the optimal \( \alpha'_{\kappa} \) s and \( \alpha_{\kappa} \)'s,
\[ \alpha'_{\kappa} = \varepsilon_{\kappa} \alpha_{\kappa} \]  \hspace{1cm} (11)
and furthermore
\[ \max \rho = \max \rho'. \]

It should be noted that the optimal combining ratios \( \beta^a_K \) and \( \beta^a_K \) are quite different to each other. Only the optimal output noise powers or the SN ratios are the same. As a special case of the Theorem, we have

**Corollary.** In a communication network, a feedback branch has the same effect as it is connected in forward direction.

This shows that the feedback branch can increase the
communication capacity of the forward direction in the case of
the continuous communication network. This fact has been made
use of to decrease the noise in a negative feedback amplifier.

Now we come to the point to solve the problem. The
optimal set of the attenuation coefficients are given by the
solution of the following adjoint electrical network. Let us
consider a bilateral network whose topological structure is
the same as the communication network and the admittance of
whose branch \( \kappa \) is \( \rho_\kappa \). When unit current is applied to
the network from the input to the output, the current
distribution \( i_\kappa \) flowing through branches \( \kappa \) is determined from
the Kirchhoff law

\[
\sum_\kappa D_\kappa^\alpha i_\kappa = 0
\]

and from the condition of minimum dissipation power, i.e., so
as to minimize

\[
\bar{W} = \sum_\kappa \frac{(i_\kappa)^2}{\rho_\kappa}
\]

It is noted that the equation of the \( i_\kappa \)'s are exactly the
same as that of the \( \alpha_\kappa \)'s. Therefore, \( \alpha_\kappa \)'s are given by
the solution of the adjoint electrical network problem whose
solution can easily be obtained by the well-known methods.
From the above considerations, we have

**Theorem 2.** The optimal SN ratio \( \rho \) is given by the two-
terminal admittance between the input and output nodes of the
adjoint network.

Proof. The power consumed in the network is \( \rho \) and is equal
to the input-output admittance, since the applied current is
1.

Obviously, the current may flow in the reverse direction
of the branch in the adjoint electrical network, unlike the
case with the communication network in which the signal can
flow only in a prescribed direction of the branch. From the
\( \alpha_\kappa \) given by the above procedure, we can determine the
optimal combining ratios \( \beta_\kappa^m \) by (5) or (6).

Even when the noises of different branches have some
correlations, the above method can also be used to determine...
the optimal $\rho^\kappa$'s. In this case, the branches of the
adjoint electrical network have mutual couplings. The
branch admittance matrix $y^{\kappa \lambda}$ of the network is determined
by

$$y^{\kappa \lambda} = \frac{n^\kappa n^\lambda}{\sqrt{P^\kappa P^\lambda}},$$

(12)

where $P^\kappa$ is the available signal power of branch $\kappa$. When
the signal powers of the branches are normalized to 1, it is
rewritten as

$$y^{\kappa \lambda} = \frac{n^\kappa n^\lambda}{n^\kappa n^\lambda}.$$  

It should be noted, however, that the theorem concerning the
inversion of a branch direction does not hold if the noise of
the branch has some correlation to the noise of another
branch.

4. Examples.

i) A feedback branch connected in parallel.

Two networks consisting of two communication channels
connected in parallel, will at first be considered. The
channels are connected
in the same direction
in one of them, and
they are connected in
reverse directions in
the other so that a
feedback branches
appears (see Fig. 2 a,b)).
Let the SN ratios of the
branches be $\rho_1$ and $\rho_2$,
respectively. Then the
optimal attenuation
coefficients $\alpha_1$ and $\alpha_2$
can be easily calculated
from the adjoint electrical network whose branch admittances
are $\rho_1$ and $\rho_2$. The current flowing in the branch 1 is

$$\alpha_1 = \frac{\rho_1}{\rho_1 + \rho_2}$$

and that in the branch 2 is

$$\alpha_2 = \frac{\rho_2}{\rho_1 + \rho_2}$$

when unit current is applied to the network. Therefore, the maximal combining ratios are calculated as

$$\beta_1 = \frac{\rho_1}{\rho_1 + \rho_2},$$

$$\beta_2 = \frac{\rho_2}{\rho_1 + \rho_2},$$

in the first network. In the latter feedback network, the branch 2 is connected in the reverse direction. Therefore, we have

$$\alpha_1 = \frac{\rho_1}{\rho_1 + \rho_2},$$

$$\alpha_2 = \frac{-\rho_2}{\rho_1 + \rho_2},$$

and then

$$\beta_1 = 1 + \frac{\rho_2}{\rho_1},$$

$$\beta_2 = -\frac{\rho_2}{\rho_1}.$$

The optimal combining ratios of the both networks are shown in Fig. 2 a) b). In the both cases, the SN ratios of the networks are the same and are given by

$$\rho = \rho_1 + \rho_2$$
It should be noted that, if the branch 2 is noiseless, i.e. \( \rho_2 \to \infty \), noiseless communication will be attainable, i.e. \( \rho \to \infty \), whether the branch is connected as a feedback branch or as parallel branch in the forward direction.

ii) A simple network shown in Fig. 4.

Find the maximal SN ratio attainable through the communication network of Fig. 4 and the maximal combining ratios of the nodes. From the adjoint electrical network having admittance \( \rho \) in branch \( \alpha \), the maximal SN ratio can be calculated as the in-cut-output 2-terminal admittance of the network. Therefore, we easily obtain

\[
\rho = \frac{10}{7}
\]

Moreover, this is invariant under the inversion of an arbitrary branches, though the optimal combining ratios are changed by an inversion.

The attenuation coefficients \( \alpha \) are calculated from the adjoint network as

\[
\begin{align*}
\alpha_1 &= \frac{5}{5}, \\
\alpha_2 &= \frac{2}{5}, \\
\alpha_3 &= \frac{1}{5}, \\
\alpha_4 &= \frac{2}{5}, \\
\alpha_5 &= \frac{3}{5},
\end{align*}
\]

Therefore, the maximal combining ratios are determined as shown in Fig. 5. When a branch, say 3, is inverted, the sign of \( \alpha_3 \) changes. The optimal combining ratios when the branch 3 is inverted, are given in Fig. 6, and those when the branch 4 is inverted are given in Fig. 7. The three networks have the same ability of signal transmission.
References


P. Elias, Channel Capacity without Coding. Appendix of the above paper.


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