A SURVEY OF THE RECENT JAPANESE INVESTIGATIONS
IN TOPOLOGICAL NETWORK THEORY AND DIAKOPTICS

by

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This material is based on a note on Graph Theory prepared for the Japanese
Arrangement Committee of the 1963 General Assembly of the URSI.
Attention is drawn to the endeavor made by Japanese investigators to
approach the essential features of network theory and diakoptics from the
standpoint of systematic methods in algebraic topology.
This first half is devoted mainly to the description of how a homological
and cohomological theory of the combinatorial structure of networks has been
constructed to clarify the definitions of trees and cotrees and their generalization
to multitrees as well as the topological features of dielectric and mag-
netic circuits.

INTRODUCTION

It may not be inappropriate to start with a reference to the mathematical and tech-
ical background of the origin of a phrase, the use of which is now fashionable. Graph
Theory seems to represent the system of applications to engineering problems of that
part of the theory of linear graphs which had been developed before the remarkable ad-
vent of homology and cohomology in Algebraic Topology. A linear graph is a one-dimen-
sional complex in the mathematician's terminology and consists of branches or 1-cells
and nodes or 0-cells. It is regarded as the mathematical expression of the engineering
term 'network'. Recently, Gabriel Kron has recommended the elimination of the termin-
ology of linear graph from the electrical network theory. He classifies three con-
licting approaches to the problems of physical-engineering networks:

1. the polyhedral theory based on multidimensional combinatorial topology that
   assumes 0-, 1-, 2-simplexes or cells,
2. the graph theory that assumes 0- and 1-simplexes,
3. the standpoint assumed by himself that admits 1-dimensional elements or
   branches alone as the elements of the network.

He assumes two engineering groups that differ with himself and that evidently belong
to 1 and 2. From the point of view of Kirchhoff's first and second laws, he empha-
sizes that neither nodes or meshes have physical significance in electrical current and
voltage analysis. So he repudiates the use of the rectangular incidence matrices such as
are thoroughly worked out in textbooks in combinatorial topology. Only a square
matrix that represents a transformation of the basis is thought to be meaningful from
his viewpoint and he calls it 'the connexion matrix'.

Of course, non-electrical applications of graphs or polyhedrons have not been
refuted by any. Not is it justified that the physics and topology of the dielectric and
magnetic features of electromagnetism need not be represented by polyhedral pictures
including 0- and 2-dimensional elements as well as the one-dimensional cells.

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In the following exposition, we shall not restrict ourselves to the graph theory. Nor shall we be concerned about the restriction of Kirchhoff's laws that is based on specific phases of the entire electromagnetism. Although it is self-evident that graph theory has an inherent connexion with computer programming as well as with project scheduling, we need not be very particular about this here because the details thereof are automatically absorbed into the more general viewpoint such as will be described in the following.

We shall begin with the recognition of the two main streams that have recently become remarkable in the application of topology to engineering.

The first is to express the questions (such as those of tree product, coding, statistical incomplete design of experiments, the Ising model, etc.) which appear sometimes trivial, in terms of linear graphs and to utilize the properties of the graphs to make the calculation simpler and easier. In regard to the basic attitude, this is little removed from the classical study of Euler's unicursal problems. Here, the graphs are sometimes means rather than the objects of the investigations. So far, attention has generally been centred on the simplified formalism, as it were, before the development of the theory of homology and cohomology, as has been said. This kind of approach does not need much preparation with regard to mathematical techniques. Investigators in this approach seem to regard the notion of algebraic chain complex as a mere mathematical concept without much significance in regard to physical and engineering problems. The concept corresponding to the domain of coefficients of the chain group is treated as an object mostly independent of the basic complex or network. Sometimes integers modulo 2 have been handled. However, little heed has been taken of their role, in physics, of their forming an additive group and as the coefficients of the chain.

The second is to endeavour to penetrate the essential combinatorial structures of engineering systems from the point of view of Algebraic Topology by extracting therefrom the algebraic properties, as chains or cochains, of the geometrical and physical quantities we are concerned about. It aims at establishing a 'Topological Theory of Networks' based on homology and cohomology theory. To what section of the general viewpoint so arrived at (to which a considerable part of the investigations from the first standpoint can be assigned) is expected to become clear in the light of the second more general approach. Whilst the general tendency overseas appears to prefer the first kind, there are some in Japan that are along the lines of the second kind among the investigations of the Research Association of Applied Geometry, (Japan).

As the theoretical structure discovered by the latter approach seems to cover a number of important practical problems, we shall present a brief survey thereof and throw light on their relationships to other individual investigations.

**PREVIOUS DEVELOPMENT**

The correspondence of the theory of electrical networks with combinatorial topology has been recognized even in the earlier works of Kirchhoff and Maxwell. But Weyl was perhaps the first to point out its close relation to modern algebraic topology. He explained the one-dimensional chain group or linear vector space having the branches as the basis vectors and the currents through them as the coefficients, from physical points of view. He introduced the concept of tree (called 'arbol' by him). It was also by him perhaps that the constraint on the branch currents (or voltages) in a cycle (or cocycle) can geometrically be treated as projection to a subspace of the vector space.

Next came W. Cauer who discussed trees and cotrees (called 'Baume' and 'Systeme unabhängig Zweigen' by him) in terms of the incidence matrix and showed that there are various quantities connected with the electrical network (such as the
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driving point immittance, transfer immittance, etc.) which can be expressed in tree-
cotree terminology.* Recently, W. S. Percival tried to solve network problems by uti-
лизing more positively the tree and 2-tree concepts with direct inspection methods.
Here, we recognized a curious, but implicative, trend in these overseas streams that
algebraic topology recedes, rather than proceeds, from Weyl to Cauer and to Percival,
leading to unicusral approach, viz., the graph theory.

As pioneers in topological network theory in Japan, we cannot fail to mention
S. Okada and Y. Miyazaki. Okada was most enthusiastic in emphasizing the importance
of topology and multidimensional linear geometry in network theory. In his work pub-
lished in conjunction with R. Onodera and H. Orui we find formulae for ex-
pressing various quantities of networks in tree-cotree terminology for the general case
that mutual coupling exists between the branches. In their theory they assign primary
importance to the concept of incidence matrices which they denote by $D^K$ (between
nodes and branches) and $R^K_p$ (between branches and meshes), where $a = 1, \ldots, m$
mean independent points or nodes $\kappa = 1, \ldots, n$ branches and $p = m + 1, \ldots, n$ independent
meshes.

They extend these matrices to include ideal transformers. The physical quantities
such as currents, voltages, impedances, etc., have been expressed by them in different
kinds of coordinates which are indexed by $a$, $\kappa$, $p$, respectively according to their kinds
and which are related to junctions, branches, meshes, trees, cotrees, etc.

Miyazaki's contribution consists, on the one hand, in his decomposition of the
impedance and/or admittance tensor into ideal basis vectors, and on the other hand, in
his original recognition of cohomological concepts in contrast to the homological con-
cepts that were in vogue, when there was still hardly a general tendency to pay attention
thoroughly to those combinatorial features of nodal type which have now become
well known.

A more formal penetration into the homological-cohomological structure was not
undertaken, however, until the younger scientists of the Research Association of Ap-
plied Geometry (Japan) became engaged in it.

THEORY OF ELECTRICAL, MAGNETIC AND DIELECTRIC NETWORKS

1. Homological-cohomological theory of electrical networks

An endeavour, meant to be thorough in connexion with the standpoint discussed
in the foregoing, to systematize the theory of networks applicable to electrical circuit
by Algebraic Topology has been made by K. Kondo and M. Iri. They start with the
obvious fact that the distribution of currents and/or voltages in the network is a
physical incarnation of the chain and/or cochain with the field of real or complex num-
bers as the coefficient domain, to put the network theory into the framework of Algebraic
Topology. Kirchhoff's laws are shown to be no other than the boundary and coboundary
relations of these chains and cochains. The homological and cohomological structure
of the network was first clarified by this systematical exposition of trees and cotrees
extended to new general concepts 'multitrees' and 'multicotrees' in extension of Percival's
2-trees. A further extension has been described by Iri. This could be regarded as
giving the foundation on which judgement should be based as to how the vernacular for-
malism so far recognized can be extended or what points thereof should further be investi-
gated. It also facilitated the application of the topological methods to problems other

* The minor determinant of the incidence matrix $D^K$ is $\pm 1$ if it corresponds to the branches of a tree, and
0 otherwise.
than electrical, as has been proved by the subsequent work of the Research Association of Applied Geometry (Japan). Although this was eventually applied to the one-dimensional practical problems, the general formulae developed do not depend on the one-dimensional restriction—that a higher cell, i.e. a branch, is incident to only two points, to one by +1, to another by −1 and non-incident to all the others (incidence number = 0). Therefore they are applicable to the theory of higher dimensions with acyclic and acyclic properties. Discussions going on elsewhere on cut-sets seem to suggest that the following fact ‘1-cocycles = 1-coboundaries’ has not been sufficiently recognized. A perfect duality is established in this theory.

The principal achievements, in more concrete terms, are as follows.

(1) On promoting an ordinary electric network to two dimensions the one-dimensional homology and cohomology groups vanish (Betti number = 0, torsion, dual torsion coefficient = 0). Therefore, the essential feature of the complex so promoted remains one-dimensional.

(2) The topological concepts such as tree, cotree, loop, cut-set, etc., the meanings of which have only vaguely been understood by the investigator’s intuition, are defined clearly by virtue of Lefschetz’s canonical basis.

(3) The relations between various technical concepts and the matrices of the network have been put in clearer terminology than hitherto, and proofs of the formulae and theorems, which have been proposed by Cauer, Okada, Onodera, have been obtained in more general form.

(4) The concepts of multitrees and multicotrees have first been introduced by dissection of the complex.

To these the following extension is added:

(5) The relation between trees, cotrees and the loop, cut-set basis related thereto are discussed in detail.

(6) The theory of multitrees and multicotrees is reduced to the theory of trees and cotrees of the cone and cocone on the given complex. Formulae for calculating the number of multitrees and that of multicotrees are obtained therefrom.

2. Minor problems in network topology

2.1 Tree product — If the branches of a network have no mutual coupling, its cut-set admittance matrix is $y_{ab} = \sum \gamma_{\lambda} D_{\lambda}^{a} D_{\lambda}^{b}$ where $\gamma_{\lambda}$s are self-admittances of the branches and $D_{\lambda}^{a}$ is the cut-set matrix and its loop impedance matrix is $z_{qp} = \sum \gamma_{\lambda} R_{\lambda}^{q} R_{\lambda}^{p} Z_{\lambda}$ where the $z_{\lambda}$s are the self-impedances and $R_{\lambda}$ is the loop matrix. The determinant $Y = \det y_{ab}$ (or $Z = \det z_{qp}$) is given by calculating the product of the self-admittances (or impedances) of all the branches of each tree (or cotree) and summing the products of all the trees (or cotrees). The situation calls for the investigation of the problem of enumerating all trees or cotrees of the given network. Therefore, there have been proposed various devices for performing the enumeration by network specialists abroad as well as in Japan as follows, although the method relying on the tree products has not been ascertained to be the best or most effective one.

(1) The methods proposed by Cauer, Okada, Onodera, and Percival are all to go over all combinations of the branches systematically and preserve the trees, if found, rejecting the non-trees.\(^5\,10\,12\) This is of course the most primitive device. However, hardly any of the other methods recently proposed are thought to be definitely superior to it. The recent proposals are mostly concerned about how to fit the calcula-
tion into the programme for electronic digital computers.

(2) The method devised by T. Fujisawa \(^{22}\) resembles the simplex method in Linear Programming. It consists in performing a series of transformations of the basis of the group of cycles from a tree to another, by replacing a branch by another. The branch is adopted for substitution or rejected by a number of criteria of the change of the primitive-loop matrix, cut-set matrix, the sign of the tree-matrix, etc.

(3) H. Watanabe \(^{23}\) proposed a method which consists in a systematical substitution of branches of a given tree to arrive at all trees of the given network. He proposed another method which is diakoptical. Its procedures resemble the inverse decomposition of a tree into multitrees by dissection studied in A-VII. He decomposes the given network into some subnetworks and the branches joining them.

(4) Y. Kasahara, K. Tezuka, Sh. Ling and T. Kitabashi \(^{24}\) published a device for obtaining all the trees mechanically when the set of primitive cut-sets (or of primitive loops) are given. We can reinterpret their method in Einstein Summation Convention and Schouten's alternation symbols \(^{25}\) as follows: The cut-sets are given by
\[ D_{K_1}^{a} \sigma_i^K (a = 1, \ldots, m) \] where \( \sigma_i^K \) are branches and are treated as vectors of a linear space. In Miyazaki's multi-vector terminology, \(^{15}\)
\[ D_{K_1}^{1} \sigma_i^{K_1} \times D_{K_2}^{2} \sigma_i^{K_2} \times \ldots \times D_{K_m}^{m} \sigma_i^{K_m} = m! D_{K_1}^{1} \ldots K_m^{m} (\sigma_i^{K_1} \times \ldots \times \sigma_i^{K_m}) \] gives the set of all trees because the coefficient \( m! D_{K_1}^{1} \ldots K_m^{m} \) on the right-hand side does not vanish if, and only if, the set of branches in the \( m \)-vector \( \sigma_i^{K_1} \times \ldots \times \sigma_i^{K_m} \) form a tree.

(5) H. Hirayama and K. Harada \(^{26}\) utilize the fact that the trees of a simplex or completely connected network are obtained by a systematical ordering of its points. They first promote the given network to a complete connexion by adding virtual branches. They then reject those trees of the complete connexion which include at least one virtual branch.

(6) The same authors, in conjunction with H. Watanabe, report a new endeavour for the 'automatic calculation of all trees in the given network' which they have tested using an electronic computer. \(^{37}\) Excepting the programme for electronic computation the method is essentially a recapitulation of Cauer, Percival, Okada and Onodera's idea to go over all conceivable sets of branches and to reject non-trees.

It is interesting that there is a tendency to return, as time goes on, to the old method such as was described in (1) \(^{9,10,12}\) in which the primary definition of 'tree' alone plays a role.

2.2 Applications of multitrees — Fujisawa \(^{22}\) discussed the application to electrical networks in a little more detail than Percival who restricted the views to 2-trees which the former did not. He introduced a concept of a generalized incidence matrix which has subsequently been proved by Iri \(^{28}\) to be the ordinary incidence matrix of the cone of the network. Fujisawa also applied the concept of multitee to a degenerate Hitchcock problem of transportation.

2.3 Network Synthesis — We first mention the synthesis of resistance networks by Okada \(^{24}\) and Onodera's generalization to the case of general branch transfer admittances or impedances. \(^{14,26}\) The aim is to require the branch impedances \( Z_{K,K'} \)'s and the loop matrix \( R^K_q \) from the given transfer admittances \( R^K_{p,q} \) \( (= R^K_p D^K_{pq} R^K_q) \). The method consists in calculating \( D^K_p \) by \( R^K_p D^K_{pq} = 0 \) for the given \( R^K_{p,q} \) and then \( R^K_p \) by \( D^K_{pq} R^K_q = 0 \). Since the synthesis in this manner needs ideal transformers and dual ideal transforms,
the incidence matrices are extended to include the turn-ratio matrices.

However, one cannot know whether the analytical solution can always correspond to a real network without ideal transformers or not. It also depends on the next problem which was solved by M. Iri who published a method of network synthesis for the given loop and/or cut-set matrix. So far, this has been performed by trial and error or by going over all the possibilities by Okada and Onodera as well as by overseas researchers. By Iri, the manipulations of the loop, or cut-set, matrix are put into correspondence with the manipulations of the network to arrive at all of the possible networks. This is regarded as a solution of the problem of finding the networks corresponding to the given homology and/or cohomology structure.

3. Networks including dielectric and magnetic circuits and the duality thereof

Our investigation in this connection starts with an extension of the topological theory of electrical networks by Y. Mizuo, M. Iri, and K. Kondo to systems characterized by torsion coefficients and Betti numbers. It was then promoted to encompass the general networks including dielectric and magnetic circuits in the topological light. These networks being essentially two-dimensional, the dielectric-magnetic duality is formally established in topological terminology. The principal achievements are:

(1) The first part is geometrical rather than physical. It originates from the need to solve the riddle that the tree determinant, when formally calculated by conventional methods, differs sometimes from +1. The analysis pointed out that the ordinary electrical networks have no one-dimensional torsion (and/or Betti) group because the specific basic meshes (2-cells) are chosen to carry this into effect. If arbitrary independent meshes are chosen as the basis, one-dimensional integral torsion groups can appear.

The puzzle has been aggravated because it disappears in the case of physical homology and/or cohomology in which the coefficient domain is a field.

(2) An ideal complex structure is defined by treating, as the expression of incidence, the linkage numbers or Kronecker indices between the meshes or nodes of the electrical network and the magnetic or dielectric elements, respectively. Torsion is formally present in this case.

(3) The turn-ratio matrix introduced by Kron, Okada and Onodera is shown not to be invariant from the point of view of the linkage whereas the linkage number of the mesh and the magnetic flux is.

(4) The duality of the system so defined (including dielectric and magnetic circuits) is shown to be the general one covering as special cases, the various mutually independent duality formalisms so far proposed by R. Julia, A. Bloch, E.C. Cherry, etc. The general duality so pointed out is not restricted to planar networks, although not covering all non-planar problems.

(To be continued.)
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Continuing the first half of this survey, which dealt with network structurology, we now attempt to review some diakoptical problems including the tearing of networks, the theory of transportation and general information networks.

DIAKOPTICS

The composite term 'Diakoptics' based on the Greek 'διακόπτω' — meaning sometimes 'back and forth' — and 'κόπτω' — meaning 'cutting into pieces' — has been suggested by Professor Philip Stanley of the Philosophy Department of Union College, Schenectady, U.S.A., to denote appropriately Dr. Gabriel Kron's approach of 'tearing a complicated physical system into small pieces, solving them separately and reconnecting them afterwards to obtain the solution of the initial system' 35. In the U.S.A., Roth's papers 36,37 have often been referred to by Kron as giving the mathematical foundations of his viewpoint. F. H. Branin's expositions 38,39 seem to be read by American and European followers of Kron. Two extensive lists of the literature including papers on diakoptics have been published by T. J. Higgins 40,41. Its application to electrical networks and equivalent circuits of physical systems has been called 'Special Diakoptics' by one of us who generalizes the term 'diakoptics' to a more general engineering thought which he calls 'A General Diakoptics' 42. On his suggestion, an information-theoretical investigation has been made by M. Iri and T. Sunaga 43 into the problem of how the labour is reduced by diakoptical manipulations. The a priori expected labour of manipulation is caught as the a priori uncertainty of the topological information in the system to be torn, defining an entropy which should be minimized.

The special-diakoptical tearing method was extended by Onodera 44,45 to include 'Codiakoptics' which is the dual of Kron's diakoptics. It consists in 'shorting' where Kron 'opens'. It fortified Kron's standpoint. Onodera studied also a multiplex diakoptics consisting in repeated applications of diakoptics or codiakoptics 44,45.

The topological foundations of the information-theoretical problem of special diakoptics have been given by S. Amari 46,47. He uses the concept of dissection which he extends from the integral theory of Ref. 19 to the field of real or complex numbers. The basic relations between the homology and cohomology groups of \( X, \bar{X}, X \), i.e., between the subsystem \( X \), the connexion system \( \bar{X} \) and the entire system \( X \) have been clarified by Alexander's duality theorem 21. In some publications on diakoptics, its topological features had been discussed using the terminology of chain-mapping. However, the

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transformation of subnetworks was the main concern. The mathematical terminology in
that connexion seems initially to mystify all who read it in a topological haze, rather
than to suggest to engineers an extension of Kron’s practical initiations. So Amari
wanted to find a general formalism for practical applications. In extending ref. 43 he
clarified the standard of the dissection to minimize the labour based on the topological
information about the network by the homology and cohomology of $X$ and $X$. He defines
the density of meshes and the density of points of the network and gives the general
principle of dissection together with a number of practical methods of dissection.

The principal achievements in ref. 46 and ref. 47 are as follows:

1. The homological and cohomological structures of $X$ (subsystem) and $X$ (connexion
   system) are clarified and a general method is developed covering both Kron’s diakoptics
   and Onodera’s codiakoptics as special cases.

2. Kron’s ‘connexion branches’ are generalized to his ‘connexion network’ $X'$ covering
   also the general case in which mutual coupling is present not only in $X$ and $X$ but also
   between them.

3. More unknown variables have been thought to be needed in diakoptics than in the
   classical node and mesh methods. Amari’s analysis has shown that such is not generally
   the case. In extreme cases the number is even reduced to half. In the example
   shown in Fig. 1, the number of variables required by:
   (i) the node method is 15,
   (ii) by the mesh method 16,
   (iii) by diakoptics, 12.

![Fig. 1]

4. The diakoptical labour has been studied from the point of view of ref. 43 to estab-
lish the principle and method of dissection which consists in a manipulation of a cer-
tain matrix carrying the topological information of the given network in its form.

**TRANSPORTATION AND INFORMATION NETWORKS**

It is because the domain of coefficients, (i.e., of the physical quantities of the
problem) is an additive group that the theory of electrical networks is fitted into alge-
braic topology as has been developed by the mathematician. However, there are many
practical engineering problems of different kinds that cannot be framed into the same
class. Consider, for example, switching circuits, transportation networks, etc., in
which physical quantities are not additive groups so that the vernacular formalism of
the mathematician’s combinatorial topology of chains and cochains

is not directly applied. A self-closed branch like Fig. 2 has meaning if
the impressed algebraic quantities do not form an additive group.
But it escapes the incidence matrix $D^K$.

![Fig. 2]

*One thought ‘It is the price of the gain by tearing’ as said by Kron. The price is indeed required only if
the dissection is performed by connexion branches alone (in Kron’s manner).
Iri studied this problem with a mathematical model of a general kind in which a
general algebraic system, other than an additive group, is impressed on the cells of the
network complex and he established a theory which is applicable to various network
problems of a considerably more general kind.\(^44\)

The general algebraic quantities are conveniently called 'informations'. They
are impressed on the cells of the network. Each 1-cell or branch is regarded as an
operator transforming and transmitting them or, conversely, each transmission or trans-
formation operation is translated into a branch. The relations between the topological
characteristics of the network and the algebraic system formed by the informations and
the branch operators have been studied. The result establishes a general theoretical
structure covering most of the network problems so far recognized in engineering science.

However, no such fine analysis based on homology and cohomology can directly
be introduced here owing to the general character of the impressed algebraic system
which is not an additive group. Only some basic concepts, such as connectedness,
loop, etc., play the principal roles in the theory. In the case where the physical quan-
tity is a Boolean ring, boundary operator $\delta$ and coboundary operator $\delta$ satisfying
$\delta^2 = 0$ ($\delta \delta = 0$) is defined and a homology cohomology terminology is meaningful.
Only a few concrete applications of the Boolean ring as the coefficient domain have
been found. For example, one may point out an application to the map-colouring prob-
lem of two-dimensional Boolean algebra consisting of four elements. The problem is
reduced to one concerning one-dimensional cycles in which we investigate whether
there is a one-cycle $C^4(\delta C^4 = 0)$ all the 1-cells of which do not vanish.

The principal achievements in ref. 48 and their applications to practical engineering
problems are as follows. Each item has been thoroughly studied and described in an
extensive monograph in RAAG Memoirs, Vol. III.

(1) The internodal characteristics of the ordinary, respectively oriented, switching
circuits have been studied by assuming, of necessity, the one-dimensional, respectively
two-dimensional, Boolean algebra as the system of the physical quantities to be im-
pressed on the branches. This has been the subject of a previous paper, which is
derived as a special application of the generalized viewpoint of ref. 48.

(2) A new method has been obtained for solving the problems of transportation net-
works in terms of a special kind of algebraic system denoted by $A$, as the physical
quantities, characterized by operations $(-)$ and $(+)$.\(^50\) \(^51\).

(3) A theory of graphs has been developed and a general algebra, the one- or two-
dimensional Boolean lattice or the ring of integers are handled in it according to the
kinds of problems concerned, as the physical quantities.\(^52\).

(4) A theory of deterministic two-person games is obtained by assuming as the im-
pressed physical quantities the elements of a two-dimensional lattice of three elements
or three-valued logical algebra.\(^53\).

(5) A new method has been arrived at for solving problems of Linear Programming of
the kind that can have a representation in terms of a network for the relation between
the variables.\(^54\). The physical quantities are summarized in a new kind of algebraic
system, which is a generalization of the above $A$ and is denoted by $A'$. The analysis
of this type is almost as simple as that of the transportation type.

MISCELLANEOUS APPLICATIONS OF THE CONCEPT OF GRAPH

The following could be mentioned as applications of graph theory other than those
which have been referred to as the principal items in the foregoing.

(1) An interesting application of the concept of graphs to the construction of a code
system with large distance has been shown by T. Kasami. The code $x$ is expressed as a one-chain over integers modulo 2 $x = (x_1, \ldots, x_n)$ where $x = 0$ or 1. Let the matrix expressing the parity check be assigned to the incidence matrix $D_\lambda^\alpha$ between the one- and zero-cells. Then, the error-free codes will emerge into the loops. If every loop consists of $p$ branches at least, any pair of loops have at least $p$ different branches from another so that the distance between the codes is always not less than $p$.

Commenting on this paper is up to the specialists of coding theory. It is regrettable that even the simplest kind of code such as Hamming’s cannot be represented by graphs so that it remains for further investigations to judge to what degree the proposed approach will reveal the general essential structure of coding.

(2) M. Kawakami and M. Segawa tried to improve the expression of ‘Signal Flow Graphs’ to arrive at a unification by elimination of complications. The incidence matrix is not used. This is an approach in parallel with some overseas tendency. There is also a paper by H. Hirayama and S. Nomura, treating the signal flow graphs as junction networks.

(3) M. Masuyama treated graph-theoretically a problem of incomplete block design in one-way and multi-way layout. He regards both the treatments and blocks as nodes of a linear graph. If a loop or tree consisting of a sequence such as ‘treatment-block-treatment-…….’ connects two treatments, their contrast can be estimated.

CONCLUSION

We have reviewed the principal items of those recent activities in which we have been involved ourselves. There are various more minor developments and further refinements, which could be properly evaluated from the viewpoint of the main stream of investigation—we hope that our comments will not only help the reader know the research activities in Japan but also serve to form a clear view of how the theory of networks and dialektics should be framed.

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desirable to prepare a formal exposition so that the matrices of the link-
age numbers can be transformed and amplified to acquire the standard
form of an incidence matrix.


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